Rayat Shikshan Sanstha's

Karmaveer Bhaurao Patil College Vashi, Navi Mumbai

Autonomous College

[University of Mumbai]

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of Course	S.Y.B.Sc. Mathematics
2	Eligibility for Admission	F.Y.B.Sc. (with Mathematics as one of the subject)
3	Passing Marks	40%
4	Ordinances/Regulations (if any)	
5	No. of Years/Semesters	One year/Two semester
6	Level	U.G.
7	Pattern	Semester
8	Status	Revised
9	To be implemented from Academic year	2019-20

AC- 02 /03 /2019
Item No-2.34

Rayat Shikshan Sanstha's
KARMAVEER BHAURAO PATIL COLLEGE, VASHI.
NAVI MUMBAI
Sector-15- A, Vashi, Navi Mumbai - 400 703

(AUTONOMOUS COLLEGE)

Syllabus for Mathematics

Program: B.Sc.

Course: S.Y.B.Sc. Mathematics

(Choice Based Credit, Grading and Semester System with effect from the academic year 2019-2020)

SEMESTER III

Course Code	UNIT	TOPICS	Credits	L/Week
MULTIVARIABLE CALCULUS				
UGMT301	I	Functions of several variables	2	3
	II	Differentiation		
	III	Applications		
ALGEBRA III				
UGMT302	I	Linear Transformations and Matrices	2	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
UGMT303	I	Permutations and Recurrence Relations	2	3
	II	Preliminary Counting		
	III	Advanced Counting		
PRACTICALS				
UGMTP03		Practical based on UGMT301, UGMT 302 and UGMT 303	3	5

SEMESTER IV

Course Code	UNIT	TOPICS	Credits	L/Week
INTEGRAL CALCULUS				
UGMT401	I	Riemann Integration	2	3
	II	Indefinite and Improper Integrals		
	III	Applications		
GROUP THEORY				
UGMT402	I	Groups and Subgroups	2	3
	II	Cyclic Groups and Cyclic subgroups		
	III	Lagrange's Theorem and Group Homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
UGMT403	I	First order First degree Differential equations	2	3
	II	Second order Linear Differential equations		
	III	Linear System of Ordinary Differential Equations		
PRACTICALS				
UGMTP04		Practical based on UGMT401, UGMT 402 and UGMT 403	3	5

Teaching Pattern for Semester III

Three lectures per week per course. Each lecture is of 48 minutes duration.
One practical (2L) per week per batch for courses UGMT301, UGMT302 combined and one practical (3L) per week for course UGMT303.
(Each batch can have maximum 25 students. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

Three lectures per week per course. Each lecture is of 48 minutes duration.
One practical (2L) per week per batch for courses UGMT401, UGMT 402 combined and one practical (3L) per week for course UGMT403.
(Each batch can have maximum 25 students. Each practical session is of 48 minutes duration.)

SEMESTER III
Course: Multivariable Calculus
Course Code: UGMT301

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)
Learning outcomes:

1. Define the Euclidean inner product and Euclidean norm function in $\mathbb{R}^n$ and find distance between two points.
2. Define open ball, open set and determine whether the given set is an open set.
3. Define scalar and vector valued functions and explain the basic results on limits and continuity of such functions.
4. Evaluate partial and directional derivative of a given function.

Content of the unit:

The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$; definition of an open subset of $\mathbb{R}^n$; neighbourhood of a point in $\mathbb{R}^n$; sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for n = 3.
Functions from $\mathbb{R}^n \to \mathbb{R}$ (scalar fields) and from $\mathbb{R}^n \to \mathbb{R}^m$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields. Directional derivatives and partial derivatives of scalar fields. Mean value theorem for derivatives of scalar fields.

Unit II: Differentiation (15 Lectures)
Learning outcomes:

1. Define differentiability, total derivative, gradient, partial derivative and higher order derivative over a scalar field.
2. State and apply chain rule of differentiability to find derivative of a composite function.
3. State and prove sufficient condition for equality of mixed partial derivative.
4. Define differentiability of a function over vector fields.

Content of the unit:

Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$; the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x,y) = x^2 + y^2$; $f(x,y,z) = x + y + z$, differentiability at a point of a function f implies continuity and existence of directional derivatives of f at the point, the existence of continuous partial derivatives in a neighborhood of a point implies differentiability at the point. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes. Chain rule for scalar fields. Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivative.

Unit III: Applications (15 lectures)
Learning outcomes:

1. Evaluate the Jacobian matrix of a vector valued function.
2. Find maxima, minima, stationary points using second derivative test in vector fields.
3. Apply chain rule of differentiation to evaluate the derivative of a composite function.

Content of the unit:

Second order Taylor's formula for scalar fields. Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, and differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only). Mean value inequality. Hessian matrix, Maxima, minima and saddle points. Second derivative test for extrema of functions of two variables. Method of Lagrange Multipliers.

Recommended Text Books:

1. T. Apostol, Calculus, Vol. 2, John Wiley.
2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

Additional Reference Books:

1. G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.
2. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.
3. Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

Course: ALGEBRA III
Course Code: UGMT302

Unit 1: Linear Transformations and Matrices (15 lectures)
Learning outcomes:

1. Define elementary and invertible matrices.
2. Perform elementary row operations and convert a given matrix to its row echelon form to compute the rank of a matrix.
3. Define linear transformations, kernel and image of a linear transformation.
4. State Rank Nullity theorem. Verify Rank Nullity theorem for a given linear transformation.
5. Define linear isomorphism and inverse of a linear isomorphism.
6. Apply Rank nullity theorem to infer if the given linear transformation is invertible.

Content of the unit:

Elementary matrices and row operations, Row Echelon form, Elementary and invertible matrices.
Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.
Linear transformations: Definition, examples, one one and onto linear transformation, Kernel and image of a linear transformation, Rank-Nullity theorem, Linear isomorphism, inverse of a linear isomorphism, Any n-dimensional real vector space is isomorphic to $\mathbb{R}^n$.

Equivalence of rank of an m× n matrix A and rank of the linear transformation $L_A: \mathbb{R}^n \to \mathbb{R}^m$ ($L_A(X) = AX$). The dimension of solution space of homogeneous system, Solution set of a non homogeneous system in terms of rank.

Unit II: Determinants (15 Lectures)
Learning outcomes:

1. Define determinant as an n-linear skew symmetric function.
2. Apply determinant to evaluate area and volume.
3. Compute the solution of $n \times n$ system of linear equations using Cramer's rule.
4. Explain linear dependence and independence of vectors using the concept of determinants.

Content of the unit:
Definition of determinant as an n- linear skew-symmetric/ alternating function. Determinant as area and volume. Existence and uniqueness of determinant function, Computation of determinant of 2×2, 3×3 matrices, diagonal matrices. Properties of determinants. Laplace expansion of a determinant, Vander monde determinant, determinant of upper triangular and lower triangular matrices.

Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, Co-factors and minors, Adjoint of an n×n matrix, Basic results such as A.adj(A) = det(A).I_n., Determinant and Invertibility, Cramer's rule.

Unit III: Inner Product Spaces (15 Lectures)
Learning outcomes:

1. Define dot product, inner product and general inner product space.
2. Define orthogonal and orthonormal sets.
3. Find orthonormal basis of a vector space using Gram-Schmidt orthogonalization process.
4. Find orthogonal projections on a line.

Content of the unit:

Dot product in $\mathbb{R}^n$; Definition of general inner product on a vector space over $\mathbb{R}$. Examples of inner product including the inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t)dt \text{ on } C[-\pi, \pi]$ the space of continuous real valued functions on $[-\pi, \pi]$ Norm of a vector in an inner product space. Cauchy-Schwartz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^2$, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^2$ and $\mathbb{R}^3$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalisation process, Simple examples in $\mathbb{R}^3$; $\mathbb{R}^4$.

Recommended Books:

1. Steven H Friedberg, Insel, Spence, Linear Algebra, Pearson Education India
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
2. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
3. L. Smith: Linear Algebra, Springer Verlag.
4. David C Lay, Linear Algebra and its applications, Pearson Education India

Course: Discrete Mathematics
Course Code: UGMT303

Unit I: Permutations and Recurrence relation (15 lectures)
Learning outcomes:

1. Define permutation and combination. State basic results on permutation.
2. Express permutations as a product of disjoint cycles.

3. Define a recurrence relation and obtain recurrence relation in counting problems.
4. Solve homogeneous and non-homogeneous recurrence relation using various methods.

Content of the unit:
Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, even and odd permutation, rank and signature of a permutation, cardinality of S_n, An.
Recurrence Relations, definition of non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous as well as non homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Unit II: Preliminary Counting (15 Lectures)
Learning outcomes:

1. Define finite, countable and uncountable sets.
2. State and prove various principles of preliminary counting.
3. Explain pigeonhole principle and its strong form and solve examples based on the principle.

Content of the unit:
Finite and infinite sets, countable and uncountable sets examples such as $\mathbb{N}, \mathbb{Z}, \mathbb{N} \times \mathbb{N}, \mathbb{Q}, (0,1), \mathbb{R}$.
Addition and multiplication Principle, counting sets of pairs, two ways counting. Stirling numbers of second kind. Simple recursion formulae satisfied by S(n, k) for k = 1, 2,…, n – 1, n
Pigeonhole principle and its strong form, its application.

Unit III: Advanced Counting (15 Lectures)
Learning outcomes:

1. Define circular permutations. Solve problems using the various formulae.
2. State principal of inclusion and exclusion and apply it to solve problems.
3. Define derangements. Solve examples using explicit formula.
4. Apply binomial and multinomial theorem in examples of counting.
5. Derive Euler's function $(\emptyset(n) : n \in \mathbb{N})$ and find $\emptyset(n)$.

Content of the unit:
Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.

$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r} \qquad \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$$

$$\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k} \qquad \sum_{i=0}^{n} \binom{n}{i} = 2^n$$

Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems. Non-negative and positive solutions of equation $x_1 + x_2 + \ldots + x_k = n$
Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\varphi(n)$.

Recommended Books:

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series: Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

UGMTP03: Practical

Suggested Practical for UGMT301

1. Sequences in $\mathbb{R}^2$ and $\mathbb{R}^3$, limits and continuity of scalar fields and vector fields, using definition and otherwise, iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/Jacobean matrix, Mean Value Inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practical for UGMT302

1. Rank-Nullity Theorem.
2. System of linear equations.
3. Determinants, calculating determinants of 2×2 matrices, $n \times n$ diagonal, upper triangular matrices using definition and Laplace expansion.
4. Finding inverses of $n\times n$ matrices using adjoint.
5. Inner product spaces, examples. Orthogonal complements in $\mathbb{R}^2$and$\mathbb{R}^3$.
6. Gram-Schmidt method.
7. Miscellaneous Theoretical Questions based on full paper

Suggested Practical for UGMT 303

1. Derangement and rank signature of permutation.
2. Recurrence relation.
3. Problems based on counting principles, Two way counting.
4. Stirling numbers of second kind, Pigeon hole principle.
5. Multinomial theorem, identities, permutation and combination of multi-set.
6. Inclusion-Exclusion principle. Euler phi function.
7. Miscellaneous theory questions from all units.

SEMESTER IV

Course: INTEGRAL CALCULUS
Course Code: UGMT401

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Riemann Integration (15 Lectures)
Learning outcomes:

1. Define Upper/Lower Riemann sums and state its properties.
2. Evaluate Upper/Lower integrals.
3. Define Riemann integral on a closed and bounded interval.
4. State and prove algebra of Riemann integrals.

Content of the unit:

Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, Additivity of Riemann integral, Algebra of Riemann integrable functions, like sum, product, modulus, Riemann integrability of monotone and continuous functions.

Unit II: Indefinite and improper integrals (15 lectures)
Learning outcomes:

1. State Fundamental theorem of integral calculus, Mean Value theorem.
2. Evaluate Integration by parts.
3. Define Improper integrals-type 1 and type 2.
4. Check convergence of improper integrals of type 1 and type 2 using Abel's or Dirichlet's test.

Content of the unit:

Fundamental theorem of integral calculus, Mean Value theorem, Integration by parts, Leibnitz rule, Improper integrals-type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests.

Unit III: Applications (15 lectures)
Learning outcomes:

1. Define Beta and gamma functions and state their properties.
2. Explain the relationship between beta and gamma functions.
3. Find Area between curves, finding volumes by slicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

Content of the unit:

Beta and gamma functions and their properties, relationship between beta and gamma functions. Applications of definite Integrals: Area between curves, finding volumes by slicing, volumes of solids of revolution-Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

References:

1. Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. T. Apostol, Calculus Vol.2, John Wiley
5. K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
6. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
7. Bartle and Sherbet, Real analysis.

Course: GROUP THEORY
Course Code: UGMT402

Unit I: Groups and Subgroups (15 Lectures)
Learning outcomes:

1. Define group, Abelian group, order of a group, centre of a group and Normalizer of a group.
2. Give examples of groups and semi groups.
3. Compute the order of elements of a group.
4. Define a subgroup and state the necessary and sufficient condition for a non-empty set to be a subgroup.
5. Determine whether a non-empty set is a subgroup of a group using one step and two step subgroup tests.

Content of the unit:

Definition of a group, Abelian group, order of a group, finite and infinite groups. Examples of groups including: $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ Under addition. $\mathbb{Q}^*(=\mathbb{Q}\{0\}), \mathbb{R}^*(=\mathbb{R}\{0\}), \mathbb{C}^*\ (=\mathbb{C}\backslash\{0\})\ \mathbb{Q}^+$(= positive rational numbers) under multiplication. $\mathbb{Z}_n$: The set of residue classes modulo n under addition, U (n): the group of prime residue classes modulo n under multiplication, The symmetric group S_n, The group of

symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for n = 3, 4), Klein 4-group, Matrix groups $M_{n\times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$; the set of invertible real matrices, under multiplication of matrices.
Properties such as

In a group $(G,.)$ the following indices rules are true for all integers n, m:

1. $a^n.a^m = a^{n+m}$ for all a in G
2. $(a^n)^m = a^{nm}$ for all a in G
3. $(a.b)^n = a^n.b^n$ for all a, b in G whenever $a * b = b * a$.

In a group $(G,.)$ the following are true:

1. The identity element e of G is unique.
2. The inverse of every element in G is unique.
3. $(a^{-1})^{-1} = a$ for all a in G
4. $(a.b)^{-1} = b^{-1}.a^{-1}$ for all a, b in G
5. If $a^2 = e$ for every a in G then (G, *) is an abelian group.
6. $(aba^{-1})^n = ab^na^{-1}$ for every a, b in G and for every integer n
7. If $(a.b)^2 = a^2.b^2$ for every a, b in G then $(G,.)$ is an abelian group.
8. $(Z_n^*,.)$ is a group if and only if n is a prime.

Properties of order of an element such as: (n and m are integers.)

1. If o(a) = n then $a^m = e$ if and only if $n|m$.
2. If o(a) = nm then $o(a^n) = m$.
3. If o(a) = n then $o(a^m) = n/(n,m)$, where (n, m) is the GCD of n and m
4. $o(aba^{-1}) = o(b)$ and o(ab) = o(ba).
5. If o(a) = n and o(b) = m, ab = ba, (n, m) = 1 then o(ab) = nm.

Subgroups: Definition, necessary and sufficient condition for a non-empty set to be a Sub-group.
The center $Z(G)$ of a group is a subgroup, Intersection of two (or a family of) subgroup is a subgroup, Union of two subgroups is not a subgroup in general; Union of two subgroups is a subgroup if and only if one is contained in the other, If H and K are subgroups of a group G then HK is a subgroup of G if and only if HK = KH.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)
Learning outcomes:

1. Define cyclic groups. Determine whether a given group is cyclic.
2. State the properties of a cyclic group.
3. Find subgroups of a cyclic group.
4. Observe that a group of prime order is cyclic.
5. List all generators, all subgroups, all generators of each subgroup of a cyclic group.

Content of the unit:

Cyclic subgroup of a group, cyclic groups, (examples including$\mathbb{Z}$; $\mathbb{Z}_n$ andμ_n).

Properties such as:

1. Every cyclic group is abelian.
2. Finite cyclic groups, infinite cyclic groups and their generators.
3. A finite cyclic group has a unique subgroup for each divisor of the order of the group.
4. Subgroup of a cyclic group is cyclic.
5. In a finite group G; $G = \langle a \rangle$ if and only if o(G) = o(a).
6. If $G = \langle a \rangle$ and o(a) = n then $G = \langle a^m \rangle$ if and only if (n, m) = 1.
7. If G is a cyclic group of order p^n and $H < G$; $K < G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)
Learning outcomes:

1. Define a coset. State Lagrange's theorem and state the corollaries of Lagrange's theorem.
2. State Euler's theorem and Fermat's theorem.
3. Define homomorphism, kernel of a homomorphism, isomorphism and automorphisms.
4. Determine whether a given map is a homomorphism, isomorphism.

Content of the unit:

Definition of Coset and properties such as:

1. If H is a subgroup of a group G and x 2 G then
2. $xH = H$ if and only if x ∈H.
3. $Hx = H$ if and only if x ∈H
4. If H is a subgroup of a group G and x; y ∈ G then
5. $xH = yH$ if and only if $x^{-1}y$ ∈H
6. $Hx = Hy$ if and only if xy^{-1}∈H

Lagrange's theorem and consequences such as Fermat's Little theorem, Euler's theorem and if a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.
Group homomorphisms and isomorphisms, automorphisms, Definition: Kernel and image of a group homomorphism. Examples including inner automorphisms.
Properties such as:
(1) $f : G \to G'$ is a group homomorphism then $ker\ f \leq G$.
(2) $f : G \to G'$ is a group homomorphism then $ker\ f = \{e\}$ if and only if f is 1-1.
(3) $f : G \to G'$ is a group homomorphism then
a) G is abelian if and only if G' is abelian.
b) G is cyclic if and only if G' is cyclic.

Recommended Books:

1. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
2. I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, Second edition.
3. Bist and Sahai, algebra, Narosa Publication.
4. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

Additional Reference Books:

1. T. W. Hungerford. Algebra, Springer.
2. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
3. I.S. Luther, I.B.S. Passi. Algebra. Vol. I and II.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
6. Combinatroial Techniques by Sharad S. Sane, Hindustan Book Agency.

Course: ORDINARY DIFFERENTIAL EQUATIONS
Course Code: UGMT403

Unit I: First order First degree Differential equations (15 Lectures)
Learning outcomes:

1. Define a differential equation and ordinary differential equation.
2. Find the order and degree of a differential equation.
3. State the existence and uniqueness theorem for first order linear differential equation.
4. Define Lipschitz function. Verify Lipschitz condition for a given function.
5. Identify different types of differential equation and solve those using appropriate methods.

Content of the unit:

Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non-linear ODE. Existence and Uniqueness Theorem for the solution of a second order initial value problem (statement only), Definition of Lipschitz function, Examples based on verifying the conditions of existence and uniqueness theorem. Review of Solution of homogeneous and non-homogeneous differential equations of first order and first degree. Notion of partial derivatives. Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors for non exact equations.

Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures)
Learning outcomes:

1. Define homogeneous and non-homogeneous second order differential equations.
2. Solve such equations using different methods based on the type.
3. Find the general solution of a homogeneous and non-homogeneous second order ordinary differential equation.

Content of the unit:

Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals. The homogeneous equation with constant coefficients. Auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Linear System of ODEs (15 Lectures)
Learning outcomes:

1. Define system of differential equations and solve the system.
2. Define and evaluate Wronskian of linear system of differential equations.
3. Determine the solution of system of homogenous and non-homogeneous equations with constant coefficient.

Content of the unit:

Existence and uniqueness theorems to be stated clearly when needed in the sequel. Study of homogeneous linear system of ODEs in two variables.

The Wronskian W (t) of two solutions of a homogeneous linear system of ODEs in two variables, Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples. System of non-homogeneous equations with constant coefficient.

Recommended Books:

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.
2. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.
3. G. F. Simmons and Steven krantz, Differential equations with applications and historical notes, McGraw Hill.
4. Dennis Zill First course in Differential equations and its applications.

UGMTP04: Practical.

Suggested Practical for UGMT401:

1. Calculation of upper sum, lower sum and Riemann integral
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.
5. Beta Gamma Functions
6. Problems on area, volume, length.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practical for UGMT402:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practical for UGMT403:

1. Solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Solving second order linear ODEs
6. Solving a system of first order linear ODES.
7. Miscellaneous Theoretical questions from all units.

Scheme of Examination

Class: S.Y.B.Sc.

I. Semester End Examinations: There will be a Semester-end Theory examination of 60 marks for each of the courses UGMT301, UGMT302, UGMT303 of Semester III and UGMT401, UGMT402, UGMT403 of semester IV to be conducted by the college.

1. Duration: The examinations shall be of 2 Hours duration.

2. Theory Question Paper Pattern:

a) There shall be **FOUR** questions. The questions first three questions shall be of **15 mark**s each based on the units I, II, III respectively. The **fourth** question shall be of **15 marks** based on the entire syllabus.

b) All the questions shall be compulsory. The questions shall have internal choices within. Including the choices, the marks for each question shall be 30.

c) The questions may be subdivided into sub-questions and the allocation of marks depends on the weightage of the topic.

II. Continuous Internal Assessment:

There will be internal evaluation of 40 marks.

<table>
<tr><th>Paper</th><th>20 Marks</th><th>10 Marks</th><th>10 Marks</th></tr>
<tr><td>Paper I</td><td>Unit Test</td><td>Assignment</td><td rowspan="3">Group Project (Max. 10 people)
Content 5 marks, Viva 5 marks
OR
Online Course (Individual)
Certificate 7 marks, Viva 3 marks</td></tr>
<tr><td>Paper II</td><td>Unit Test</td><td>Assignment</td></tr>
<tr><td>Paper III</td><td>Unit Test</td><td>Assignment</td></tr>
</table>

Question paper pattern for Unit Test of 20 marks:

The unit test for 20 marks will be conducted online. There shall be 20 compulsory multiple choice questions with single correct answer, each carrying one mark.

III. Semester End Examinations Practical:

At the end of the Semesters III and IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses **UGMTP03**, **UGMTP04**.

In semester III, the Practical examinations for **UGMT301** and **UGMT302** are held together and the Practical examination for **UGMT303** is held separately

In semester IV, the Practical examinations for **UGMT401** and **UGMT402** are held together and the Practical examination for **UGMT403** is held separately.

Paper pattern: The question paper shall have three parts A, B, C.

Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions.

(8×3 = 24 Marks)

Section II Problems: Attempt any Two out of Three. (8×2 = 16 Marks)

Practical Course	Part A	Part B	Part C	Marks out of	Duration
UGMTP03	Questions from UGMT301	Questions from UGMT302	Questions from UGMT303	120	3 hours
UGMTP04	Questions from UGMT401	Questions from UGMT402	Questions from UGMT403	120	3 hours

Marks for Journals and Viva:

For each course UGMT301, UGMT302, UGMT303, UGMT401, UGMT402 and UGMT403:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.